

Fresnel analysis of the wave propagation in nonlinear electrodynamics

Yuri N. Obukhov*

Instituto de Física Teórica, UNESP, Rua Pamplona 145, 01405-900 São Paulo, SP, Brazil

Guillermo F. Rubilar

Institute for Theoretical Physics, University of Cologne, 50923 Köln, Germany

Abstract

We study the wave propagation in nonlinear electrodynamical models. Particular attention is paid to the derivation and the analysis of the Fresnel equation for the wave covectors. For the class of general nonlinear Lagrangian models, we demonstrate how the originally quartic Fresnel equation factorizes, yielding the generic birefringence effect. We show that the closure of the effective constitutive (or jump) tensor is necessary and sufficient for the absence of birefringence, i.e., for the existence of a unique light cone structure. As another application of the Fresnel approach, we analyze the light propagation in a moving isotropic nonlinear medium. The corresponding effective constitutive tensor contains non-trivial skewon and axion pieces. For nonmagnetic matter, we find that birefringence is induced by the nonlinearity, and derive the corresponding optical metrics.

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*On leave from: Department of Theoretical Physics, Moscow State University, 117234 Moscow, Russia

1. INTRODUCTION

Wave phenomena belong to the most interesting and important processes in physics. Among other field theories, nonlinear electrodynamics attracts much attention in connection with the prominent role played by light in the experimental and theoretical studies of the structure of spacetime and matter.

Nonlinearities in electrodynamical models can arise in different ways in classical and quantum field theories. For example, the old Born-Infeld theory [1] was a fundamental theory alternative to the classical Maxwell electrodynamics which provided a model of a classical electron. On the other hand, quantum Maxwell electrodynamics predicts nonlinear effects which arise due to the radiative corrections, see [2–4]. Finally, in the modern string theories a generalized Born-Infeld action naturally arises as the leading part of the effective string action, see [5–7], for example.

Wave propagation in the various nonlinear electrodynamical theories was studied previously in [8–12] and also in [13–16]. A general feature revealed in these studies is the existence of birefringence. In crystal optics, the notion of birefringence means the emergence of two rays (ordinary and extraordinary) with different velocities inside the material medium. We will use the expression “birefringence” in a similar sense, associating it with the situation when two different light cones exist for the wave normal covectors. However, the earlier results are incomplete in the sense that the full Fresnel equation, governing the wave normals, was never derived explicitly. Moreover, it was not demonstrated how it happens that the original quartic surface of wave normals reduces to the light cone. That is the primary interest in our study, and we will try to clarify this aspect for the nonlinear electrodynamical models, using and expanding our earlier results obtained within the framework of linear electrodynamics [21,23].

Our basic tool will be the *general formula* for the Fresnel equation derived earlier within linear electrodynamics. Now we observe that the analysis of the wave propagation in a general nonlinear model reduces to the linear case because the jumps of the derivatives of

the excitation and of the field strength are in all cases related by a linear law. We can make then use of our master formula for the Fresnel tensor and *derive the Fresnel equation for any nonlinear model*, and thereby explain the reduction to the light cones.

2. ELECTROMAGNETIC WAVES AND FRESNEL TENSOR

Quite generally, Maxwell's equations for the excitation 2-form $H = (\mathcal{D}, \mathcal{H})$ and the field strength 2-form $F = (E, B)$ read

$$dH = J, \quad dF = 0. \quad (2.1)$$

Here J is the electric current 3-form. These equations must be supplemented by a constitutive law $H = H(F)$. The latter relation contains the crucial information about the underlying physical continuum (i.e., spacetime and/or material medium). Mathematically, this constitutive law arises either from a suitable phenomenological theory of a medium or from the electromagnetic field Lagrangian. It can be a nonlinear or even nonlocal relation between the electromagnetic excitation and the field strength.

If local coordinates x^i are given, with $i, j, \dots = 0, 1, 2, 3$, we can decompose the excitation and field strength 2-forms into their components according to

$$H = \frac{1}{2} H_{ij} dx^i \wedge dx^j, \quad F = \frac{1}{2} F_{ij} dx^i \wedge dx^j. \quad (2.2)$$

We will study the propagation of a discontinuity of the electromagnetic field following the lines of Ref. [21], see also Refs. [23]. The surface of discontinuity S is defined locally by a function Φ such that $\Phi = \text{const}$ on S . Across S , the geometric Hadamard conditions are satisfied:

$$[F_{ij}] = 0, \quad [\partial_i F_{jk}] = q_i f_{jk}, \quad (2.3)$$

$$[H_{ij}] = 0, \quad [\partial_i H_{jk}] = q_i h_{jk}. \quad (2.4)$$

Here $[\mathcal{F}](x)$ denotes the discontinuity of a function \mathcal{F} across S , and $q_i := \partial_i \Phi$ is the wave covector. Given the constitutive law $H(F)$, which determines the excitation in terms of the

field strength, the corresponding tensors f_{ij} and h_{ij} , describing the jumps of the derivatives of field strength and excitation, are related by [24]

$$h_{ij} = \frac{1}{2} \kappa_{ij}{}^{kl} f_{kl}, \quad \text{with} \quad \kappa_{ij}{}^{kl} := \frac{\partial H_{ij}}{\partial F_{kl}}. \quad (2.5)$$

We will call $\kappa_{ij}{}^{kl}$ the *jump tensor*. In linear electrodynamics, its components coincide with the components of the constitutive tensor (which describes the linear law $H_{ij} = \frac{1}{2} \kappa_{ij}{}^{kl} F_{kl}$), and they are independent of the electromagnetic field. However, in general the jump tensor $\kappa_{ij}{}^{kl}$ is a function of the electromagnetic field, the velocity of matter, the temperature, and other physical and geometrical variables. Quite remarkably, all the earlier results obtained for linear electrodynamics remain also valid in the general case because whatever local relation $H(F)$ may exist, the relation between the *jumps* of the field derivatives, according to (2.5), is always linear.

If we use Maxwell's equations (2.1), then (2.3) and (2.4) yield

$$\epsilon^{ijkl} q_j h_{kl} = 0, \quad \epsilon^{ijkl} q_j f_{kl} = 0. \quad (2.6)$$

Let us introduce the analog of the conventional constitutive matrix

$$\chi^{ijkl} := \frac{1}{2} \epsilon^{ijmn} \kappa_{mn}{}^{kl} = \frac{\partial \mathcal{H}^{ij}}{\partial F_{kl}}, \quad (2.7)$$

where we denote $\mathcal{H}^{ij} := \frac{1}{2} \epsilon^{ijmn} H_{mn}$. Similarly to $\kappa_{ij}{}^{kl}$, we will often call the tensor χ^{ijkl} the *jump tensor density*.

Now, making use of (2.5) and (2.7), we rewrite the system (2.6) as

$$\chi^{ijkl} q_j f_{kl} = 0, \quad \epsilon^{ijkl} q_j f_{kl} = 0. \quad (2.8)$$

Solving the last equation by $f_{ij} = q_i a_j - q_j a_i$, we finally reduce (2.8)₁ to

$$\chi^{ijkl} q_j q_k a_l = 0. \quad (2.9)$$

This algebraic system has a nontrivial solution for a_i only when the wave covectors satisfy a certain condition. The latter gives rise to our *covariant Fresnel equation* [21,23]

$$\mathcal{G}^{ijkl}(\chi) q_i q_j q_k q_l = 0, \quad (2.10)$$

with the fourth order Fresnel tensor density \mathcal{G} of weight +1 defined by

$$\mathcal{G}^{ijkl}(\chi) := \frac{1}{4!} \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \chi^{mnr(i} \chi^{j|ps|k} \chi^{l)qtu}. \quad (2.11)$$

It is totally symmetric, $\mathcal{G}^{ijkl}(\chi) = \mathcal{G}^{(ijkl)}(\chi)$, and thus has 35 independent components.

3. NONLINEAR ELECTRODYNAMICS

Let us denote the two independent electromagnetic invariants as

$$I_1 := F_{ij} F^{ij}, \quad I_2 := F_{ij} \tilde{F}^{ij}, \quad (3.1)$$

where $\tilde{F}^{ij} = \frac{1}{2} \eta^{ijkl} F_{kl}$ and $\eta^{ijkl} := |g|^{-1/2} \epsilon^{ijkl}$. The Hodge operator for the exterior forms is denoted by the star $*$, as usual; but we will use a tilde to denote the dual 2-tensors. We will not restrict ourselves to the case of Minkowski spacetime with $g_{ij} = \text{diag}(1, -1, -1, -1)$, but instead g_{ij} will be considered as an arbitrary curved Lorentzian spacetime metric.

The class of nonlinear electrodynamics models we study are described, in general, by the Lagrangian 4-form

$$V = L \eta, \quad \text{with} \quad L = L(I_1, I_2). \quad (3.2)$$

Here, as usual, η is the 4-form of the spacetime volume. The electromagnetic excitation 2-form, which enters the Maxwell equation (2.1), is derived as the derivative of the Lagrangian form, $H = -\partial V / \partial F$. Explicitly, we then have the nonlinear constitutive law

$$H = 4 (-L_1 *F + L_2 F). \quad (3.3)$$

We denote the partial derivatives of the Lagrangian function w.r.t. its arguments as

$$L_a := \frac{\partial L}{\partial I_a}, \quad L_{ab} := \frac{\partial^2 L}{\partial I_a \partial I_b}, \quad a, b = 1, 2. \quad (3.4)$$

In accordance with (2.5) and (2.7), the direct differentiation of (3.3) yields the jump tensor

$$\chi^{ijkl} = \sqrt{|g|} \left[k_1 g^{i[k} g^{l]j} + k_2 F^{ij} F^{kl} + k_3 \tilde{F}^{ij} F^{kl} + k_4 F^{ij} \tilde{F}^{kl} + k_5 \tilde{F}^{ij} \tilde{F}^{kl} + k_6 \eta^{ijkl} \right]. \quad (3.5)$$

The coefficients k_A , $A = 1, \dots, 6$, are functions of the electromagnetic fields:

$$k_1 = 4L_1, \quad k_2 = 8L_{11}, \quad k_3 = k_4 = 8L_{12}, \quad k_5 = 8L_{22}, \quad k_6 = 2L_2. \quad (3.6)$$

The identifications (3.6) are derived for the nonlinear Lagrangian (3.2) from the constitutive law (3.3). However, in most computations below we will consider the most general case with unspecified arbitrary coefficients k_A . This may be useful if we want to study the nonlinear electrodynamics of a more general type, for instance, with the dissipation effects and/or in moving media.

In general, the untwisted tensor density $\chi^{ijkl}(x)$ of weight $+1$ has 36 independent components. We can decompose it into irreducible pieces [23] with respect to the 6-dimensional (“bivector”) linear group as follows:

$$\chi^{ijkl} = {}^{(1)}\chi^{ijkl} + {}^{(2)}\chi^{ijkl} + {}^{(3)}\chi^{ijkl}. \quad (3.7)$$

The irreducible pieces of χ are defined by

$${}^{(2)}\chi^{ijkl} := \frac{1}{2} \left(\chi^{ijkl} - \chi^{klij} \right) = -{}^{(2)}\chi^{klij}, \quad {}^{(3)}\chi^{ijkl} := \chi^{[ijkl]}, \quad (3.8)$$

$${}^{(1)}\chi^{ijkl} := \chi^{ijkl} - {}^{(2)}\chi^{ijkl} - {}^{(3)}\chi^{ijkl} = {}^{(1)}\chi^{klij}. \quad (3.9)$$

The irreducible pieces ${}^{(1)}\chi$, ${}^{(2)}\chi$, and ${}^{(3)}\chi$ have 20, 15, and 1 independent components, respectively. The possible presence of an axion piece ${}^{(3)}\chi$ was first studied by Ni [25], whereas a constitutive law with an isotropic skewon ${}^{(2)}\chi$ was discussed by Nieves and Pal [26].

In the Lagrangian models (3.2), the effective constitutive tensor is automatically *symmetric*, i.e. ${}^{(2)}\chi = 0$, which follows from (3.6), since $k_3 = k_4$. However, in general the jump tensor (3.5) has all the three irreducible pieces:

$${}^{(1)}\chi^{ijkl} = \sqrt{|g|} \left[k_1 g^{i[k} g^{l]j} + k_2 F^{ij} F^{kl} + \frac{(k_3 + k_4)}{2} \left(\tilde{F}^{ij} F^{kl} + F^{ij} \tilde{F}^{kl} \right) \right]$$

$$+ k_5 \tilde{F}^{ij} \tilde{F}^{kl} - \frac{1}{12} [(k_3 + k_4)I_1 + (k_5 - k_2)I_2] \eta^{ijkl} \Big], \quad (3.10)$$

$$^{(2)}\chi^{ijkl} = \frac{(k_3 - k_4)}{2} \sqrt{|g|} \left(\tilde{F}^{ij} F^{kl} - F^{ij} \tilde{F}^{kl} \right), \quad (3.11)$$

$$^{(3)}\chi^{ijkl} = \frac{1}{12} \sqrt{|g|} [(k_3 + k_4)I_1 + (k_5 - k_2)I_2 + 12k_6] \eta^{ijkl}. \quad (3.12)$$

4. FRESNEL EQUATION AND BIREFRINGENCE

Our study of the algebraic system in the framework of the Hadamard formalism yields the Fresnel equation in the generally covariant form (2.10) with the Fresnel tensor density (2.11). For the explicit jump tensor density (3.5), it thus remains to substitute its components into (2.11). A straightforward calculation yields the result:

$$\mathcal{G}^{ijkl} = -\frac{k_1}{8} \sqrt{|g|} \left(\mathcal{X} g^{(ij} g^{kl)} + 2\mathcal{Y} g^{(ij} t^{kl)} + \mathcal{Z} t^{(ij} t^{kl)} \right). \quad (4.1)$$

Here we denote

$$t^{ij} := F^{ik} F^j_k, \quad (4.2)$$

and

$$\mathcal{X} = k_1^2 + \frac{k_1}{2} (k_3 + k_4) I_2 - k_1 k_5 I_1 + \frac{1}{4} (k_3 k_4 - k_2 k_5) I_2^2, \quad (4.3)$$

$$\mathcal{Y} = k_1 (k_2 + k_5) + (k_3 k_4 - k_2 k_5) I_1, \quad (4.4)$$

$$\mathcal{Z} = 4 (k_2 k_5 - k_3 k_4). \quad (4.5)$$

The most remarkable property of (4.1) is that it is obviously factorizable into a product of 2 second order tensors. Correspondingly, the quartic Fresnel surface of the wave normals reduces to the product of two second order surfaces:

$$\mathcal{G}^{ijkl}(\chi) q_i q_j q_k q_l = \frac{-k_1}{8\mathcal{X}} (g_1^{ij} q_i q_j) (g_2^{kl} q_k q_l) = \frac{-k_1}{8\mathcal{Z}} (\bar{g}_1^{ij} q_i q_j) (\bar{g}_2^{kl} q_k q_l). \quad (4.6)$$

In other words, the wave normals lie not on the quartic surface but on one of the two cones which are determined by the pair of *optical metric* tensors:

$$g_1^{ij} := \mathcal{X} g^{ij} + (\mathcal{Y} + \sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}) t^{ij}, \quad (4.7)$$

$$g_2^{ij} := \mathcal{X} g^{ij} + (\mathcal{Y} - \sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}) t^{ij}. \quad (4.8)$$

The second equality in (4.6) offers a different description of the cones by means of the conformally equivalent metric tensors:

$$\bar{g}_1^{ij} := (\mathcal{Y} - \sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}) g^{ij} + \mathcal{Z} t^{ij} = \frac{1}{\mathcal{X}} (\mathcal{Y} - \sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}) g_1^{ij}, \quad (4.9)$$

$$\bar{g}_2^{ij} := (\mathcal{Y} + \sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}) g^{ij} + \mathcal{Z} t^{ij} = \frac{1}{\mathcal{X}} (\mathcal{Y} + \sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}) g_2^{ij}. \quad (4.10)$$

Thus, the general Fresnel analysis demonstrates that in *any* nonlinear electrodynamics model (3.2) the quartic wave surface *always* reduces to two light cones. This is the birefringence effect which is thus a general feature of the nonlinear electrodynamics.

5. PROPERTIES OF OPTICAL METRICS

Let us discuss the results obtained in the previous section. The following general observations are in order.

The Fresnel equation is trivially satisfied *for all* wave covectors when $k_1 = 0$, see (4.1). Thus, – in order to have waves – every electrodynamic Lagrangian L *should necessarily* depend on the invariant $I_1 = F_{ij} F^{ij}$ (thus providing $k_1 \neq 0$).

Accordingly, we will always assume that $k_1 \neq 0$.

In order to have a decent light propagation, the optical metrics should be real and with Lorentzian signature. How can one be a priori sure that for every L an optical metric necessarily has these properties?

Using (4.3)-(4.5) we find an explicit expression for the quantity under the square root in the above formulas :

$$\mathcal{Y}^2 - \mathcal{X}\mathcal{Z} = N_1^2 + N_2 N_3, \quad (5.1)$$

where we have denoted

$$N_1 := k_1 (k_2 - k_5) + (k_3 k_4 - k_2 k_5) I_1, \quad (5.2)$$

$$N_2 := 2k_1 k_3 + (k_3 k_4 - k_2 k_5) I_2, \quad (5.3)$$

$$N_3 := 2k_1 k_4 + (k_3 k_4 - k_2 k_5) I_2. \quad (5.4)$$

The expression (5.1) is *always non-negative* in every nonlinear theory (3.2) because $N_2 = N_3$ when we take into account that $k_3 = k_4$, see (3.6).

The signature of a four-dimensional metric is Lorentzian if and only if its determinant is negative. Straightforward computation yields for the optical metrics (4.7)-(4.8):

$$\left(\det g_a^{ij} \right) = \left(\det g^{ij} \right) \left[\alpha^2 + \frac{\alpha \beta_a}{2} I_1 + \text{sign}(g) \frac{\beta_a^2}{16} I_2^2 \right]^2, \quad a = 1, 2. \quad (5.5)$$

Here $\alpha = \mathcal{X}$ and $\beta_1 = \mathcal{Y} + \sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}$, $\beta_2 = \mathcal{Y} - \sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}$. As we see, both optical metrics have Lorentzian signature as soon as the spacetime metric g^{ij} is Lorentzian.

Summarizing, we have demonstrated that (4.7)-(4.8) indeed describe the generic effect of a birefringent light propagation for *all* nonlinear Lagrangians.

Recently, the emergence of the two “effective geometries” has been described in [13,14] without using the Fresnel approach. This result is in a qualitative agreement with our analysis. In order to prove the quantitative correspondence, one needs to show that our optical metrics are conformally equivalent to the effective metrics of [13,14]. Although the corresponding comparison is rather complicated and the direct proof is still not available, one can verify that $\sqrt{\mathcal{Y}^2 - \mathcal{X}\mathcal{Z}}$ is equal to $64\sqrt{\Delta}$ of [13]. Moreover, one can recast the optical metrics (4.7)-(4.10) into the form of the so-called Boillat metrics of [15].

6. SPECIAL LAGRANGIANS

It is worthwhile to study in a greater detail certain particular nonlinear models which are potentially of physical interest.

A. Lagrangian $L = L(I_2)$

When the Lagrangian depends only on the second electromagnetic invariant, we have $k_1 = 0$, and there are no waves in such models.

B. Lagrangian $L = L(I_1)$

For the Lagrangian which, on the contrary, depends on the first invariant only, we find $k_3 = k_4 = k_5 = 0$. Accordingly, from (4.3)-(4.5) we find $\mathcal{X} = k_1^2$, $\mathcal{Y} = k_1 k_2$, $\mathcal{Z} = 0$ and thus (4.7)-(4.8) yield

$$g_1^{ij} = k_1 (k_1 g^{ij} + 2k_2 t^{ij}), \quad g_2^{ij} = k_1^2 g^{ij}. \quad (6.1)$$

Correspondingly, we still have birefringence with some photons moving along the standard null rays of the spacetime metric g^{ij} , whereas other photons choosing the rays null with respect to the optical metric $L_1 g^{ij} + 4L_{11} t^{ij}$, cf. [13,14].

C. Lagrangian $L = U(I_1) + \alpha I_2$

This is a simple generalization of the above case. Here α does not depend on the electromagnetic field, although it is not a constant, in general. When it depends on the spacetime coordinates, $\alpha = \alpha(x)$, one can identify it with the axion field.

Here we again have $k_3 = k_4 = k_5 = 0$ and we recover the same light cone structure (6.1). To put it differently, axion does not disturb the light cones which are solely determined by the spacetime metric and by the dependence of the Lagrangian on the invariant I_1 .

D. Lagrangian $L = a I_1 + V(I_2)$

Then $k_2 = k_3 = k_4 = 0$ which yields $\mathcal{X} = k_1^2 - k_1 k_5 I_1$, $\mathcal{Y} = k_1 k_5$, $\mathcal{Z} = 0$. Consequently,

$$g_1^{ij} = k_1 [(k_1 - k_5 I_1) g^{ij} + 2k_5 t^{ij}], \quad g_2^{ij} = k_1 (k_1 - k_5 I_1) g^{ij}, \quad (6.2)$$

i.e., there is again birefringence with one cone determined by the standard spacetime metric.

E. Born-Infeld theory

The Lagrangian of the Born-Infeld (BI) theory [1] reads:

$$L = b^2 \left(\sqrt{1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2} - 1 \right). \quad (6.3)$$

Here b is the coupling constant. By differentiation, we find:

$$k_1 = 4L_1 = \frac{1}{\left(1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2\right)^{1/2}}, \quad (6.4)$$

$$k_2 = 8L_{11} = \frac{-1}{2b^2 \left(1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2\right)^{3/2}}, \quad (6.5)$$

$$k_3 = k_4 = 8L_{12} = \frac{I_2}{8b^4 \left(1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2\right)^{3/2}}, \quad (6.6)$$

$$k_5 = 8L_{22} = \frac{-\left(1 + \frac{1}{2b^2} I_1\right)}{2b^2 \left(1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2\right)^{3/2}}. \quad (6.7)$$

Correspondingly,

$$\mathcal{X} = \frac{\left(1 + \frac{1}{2b^2} I_1\right)^2}{\left(1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2\right)^2}, \quad (6.8)$$

$$\mathcal{Y} = \frac{-\left(1 + \frac{1}{2b^2} I_1\right)}{b^2 \left(1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2\right)^2}, \quad (6.9)$$

$$\mathcal{Z} = \frac{1}{b^4 \left(1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2\right)^2}. \quad (6.10)$$

As a result, we find $\mathcal{Y}^2 - \mathcal{X}\mathcal{Z} = 0$, and thus the two optical metrics (4.7)-(4.8) coincide,

$$g_1^{ij} = g_2^{ij} = \frac{\left(1 + \frac{1}{2b^2} I_1\right)}{\left(1 + \frac{1}{2b^2} I_1 - \frac{1}{16b^4} I_2^2\right)^2} \left[\left(1 + \frac{1}{2b^2} I_1\right) g^{ij} - \frac{1}{b^2} t^{ij} \right]. \quad (6.11)$$

Birefringence disappears, and the photons propagate along a single light cone determined by the “quasi-metric” of Plebanski [11]

$$\left(1 + \frac{1}{2b^2} I_1\right) g^{ij} - \frac{1}{b^2} t^{ij}. \quad (6.12)$$

7. NO BIREFRINGENCE CONDITION

In this section, we will restrict our attention to the Lagrangian theories for which the constitutive tensor is (3.5), and the coefficients are derived as (3.6). It is important that the Fresnel analysis reveals that $k_1 \neq 0$, otherwise there is no decent wave propagation at all.

As it is clear from (5.1), the necessary and sufficient condition of the absence of birefringence is provided by the pair of equations:

$$N_1 = k_1(k_2 - k_5) + (k_3k_4 - k_2k_5)I_1 = 0, \quad (7.1)$$

$$N_2 = N_3 = 2k_1k_3 + (k_3k_4 - k_2k_5)I_2 = 0. \quad (7.2)$$

Here the property $k_3 = k_4$ of the Lagrangian models is used.

Taking into account that all k 's are the partial derivatives of the Lagrangian L w.r.t. I_1 and/or I_2 as displayed in (3.6), we can view the above system as a pair of partial differential equations, the solution $L = L(I_1, I_2)$ of which describes a *model without birefringence* (i.e., with a single light cone). At least two such particular solutions are already known: one is rather simple, namely, the standard Maxwell theory with $L = I_1/4$. Another is more nontrivial – this is the Born-Infeld theory with the Lagrangian (6.3). One may ask the question: Are these the only solutions of the system (7.1)-(7.2)? The immediate inspection of the system (7.1)-(7.2) shows that the answer is negative. For example, the Lagrangian function $L(I_1, I_2) = aI_1/I_2$, with constant a , satisfies the equations (7.1)-(7.2). Such a nonlinear (and nonpolynomial) model thus also has no birefringence. It is an open problem to find the complete set of solutions of (7.1)-(7.2), leading then to a single light cone.

8. CLOSURE CONDITION

Let us denote the “traceless” part of the jump tensor as

$$\chi^{ijkl} := {}^{(1)}\chi^{ijkl} + {}^{(2)}\chi^{ijkl}. \quad (8.1)$$

As we know [23], only the traceless part determines the Fresnel surface, whereas the axion part ${}^{(3)}\chi^{ijkl}$ drops out completely from the wave propagation analysis.

For the general jump tensor (3.5), we find:

$$\begin{aligned} \frac{1}{4} \hat{\epsilon}_{pqij} \chi^{ijkl} \hat{\epsilon}_{klrs} \chi^{rsmn} = & \left(-k_1^2 + \frac{a_0^2}{k_1^2} \right) \delta_p^{[m} \delta_q^{n]} + a_0 \eta_{pq}{}^{mn} \\ & + a_1 F_{pq} \tilde{F}^{mn} + a_2 \tilde{F}_{pq} F^{mn} + a_3 F_{pq} F^{mn} + a_4 \tilde{F}_{pq} \tilde{F}^{mn}. \end{aligned} \quad (8.2)$$

Here the coefficients read:

$$a_0 = \frac{k_1}{6} [(k_3 + k_4) I_1 + (k_5 - k_2) I_2] = \frac{1}{6} \left[-N_1 I_2 + \frac{(N_2 + N_3)}{2} I_1 \right], \quad (8.3)$$

$$\begin{aligned} a_1 = & -(k_3 + k_4) k_1 - k_3 k_4 I_2 + \frac{k_5}{3} [2(k_3 + k_4) I_1 + (2k_5 + k_2) I_2] \\ = & N_1 \left(-\frac{2k_5}{3k_1} I_2 \right) + \frac{(N_2 + N_3)}{2} \left(-1 + \frac{2k_5}{3k_1} I_1 \right), \end{aligned} \quad (8.4)$$

$$\begin{aligned} a_2 = & -(k_3 + k_4) k_1 - k_3 k_4 I_2 + \frac{k_2}{3} [2(k_3 + k_4) I_1 + (k_5 + 2k_2) I_2] \\ = & N_1 \left(\frac{2k_2}{3k_1} I_2 \right) + \frac{(N_2 + N_3)}{2} \left(-1 - \frac{2k_2}{3k_1} I_1 \right), \end{aligned} \quad (8.5)$$

$$\begin{aligned} a_3 = & (k_5 - k_2) k_1 + k_2 k_5 I_1 + \frac{k_3}{3} [2(k_5 - k_2) I_2 + (2k_3 - k_4) I_1] \\ = & N_1 \left(-1 - \frac{2k_3}{3k_1} I_2 \right) + \frac{(N_2 + N_3)}{2} \left(\frac{2k_3}{3k_1} I_1 \right), \end{aligned} \quad (8.6)$$

$$\begin{aligned} a_4 = & -(k_5 - k_2) k_1 - k_2 k_5 I_1 - \frac{k_4}{3} [2(k_5 - k_2) I_2 + (2k_4 - k_3) I_1] \\ = & N_1 \left(1 + \frac{2k_4}{3k_1} I_2 \right) + \frac{(N_2 + N_3)}{2} \left(-\frac{2k_4}{3k_1} I_1 \right). \end{aligned} \quad (8.7)$$

The second lines in (8.4)-(8.7) give the a 's in terms of the combinations (7.1)-(7.2). Certainly, we use the assumption that $k_1 \neq 0$.

Like the constitutive tensor of linear electrodynamics, the jump tensor $\kappa_{ij}{}^{kl} = \frac{1}{2} \hat{\epsilon}_{ijmn} \chi^{mnkl}$ determines a linear map in the 6-dimensional space of 2-forms. When the action of this map, repeated twice, brings us (up to a factor) back to the identity map, we speak of the *closure* property of $\kappa_{ij}{}^{kl}$. The importance of the closure property is related to the fact that ultimately $\kappa_{ij}{}^{kl}$ turns out to be the *duality operator* which determines a unique conformal Lorentzian metric on the spacetime.

In nonlinear electrodynamics, the jump tensor (3.5) has the closure property when

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = 0, \quad a_3 = 0, \quad a_4 = 0, \quad (8.8)$$

as is evident from (8.2).

9. EQUIVALENCE OF CLOSURE AND NO BIREFRINGENCE CONDITIONS

In linear electrodynamics, there is much evidence (although the final rigorous proof is still missing) that the quartic Fresnel surface of wave covectors reduces to a unique light cone if and only if the constitutive tensor has the closure property.

For the nonlinear Lagrangian theories it is possible to make some progress in solving the equivalence problem. In a certain sense, the situation here is simpler because we have discovered that the quartic Fresnel surface is always reduced to the product of the light cones (birefringence). The next step is thus to study under which conditions the birefringence disappears and, correspondingly, a unique light cone arises.

Theorem: In the general nonlinear electrodynamical model described by the Lagrangian (3.2), the Fresnel equation implies a single light cone (no birefringence) if and only if the traceless part of the jump tensor satisfies the closure property.

Proof: As a preliminary remark, we note that for the Lagrangian models (3.2) the jump tensor (3.5) is symmetric because $k_3 = k_4$. As a result, $N_2 = N_3$.

The necessary condition is evident. The birefringence is absent when $N_1 = N_2 (= N_3) = 0$, see (7.2). Then we immediately read from (8.3)-(8.7) that $a_0 = a_1 = a_2 = a_3 = a_4 = 0$, and thus the closure is recovered from (8.2).

The sufficient condition is also proved straightforwardly. The closure condition (8.8) has the unique solution

$$N_1 = 0, \quad \frac{N_2 + N_3}{2} = 0, \quad (9.1)$$

when we analyze (8.3)-(8.7). Since for the Lagrangian models we have $N_2 = N_3$, then (9.1) yields $N_1 = N_2 = N_3 = 0$. Thus, there is no birefringence.

To put it differently, we have proven that the closure of the traceless jump tensor is the necessary and sufficient condition for the reduction of the fourth order Fresnel wave surface to a single light cone. This is true for *all nonlinear* Lagrangian electrodynamical theories

(3.2). Returning to our studies of the general *linear* electrodynamics, we expect that a similar result holds true there.

A. On asymmetric jump tensors

Symmetry of the jump tensor is very important in the equivalence proof above. In order to clarify this point, let us consider an arbitrary jump tensor (3.5) without assuming the explicit form of the coefficients (3.6). When $k_3 \neq k_4$, the jump tensor has the nontrivial skewon part (3.11). Also, $N_2 \neq N_3$ [in fact, as we can see from (5.3) and (5.4), $N_2 - N_3 = 2k_1(k_3 - k_4)$].

We can easily verify that the closure of an asymmetric operator is not equivalent to the no-birefringence property. Indeed, take, for instance, $k_1 \neq 0$, $k_2 = k_3 = 0$, $k_4 \neq 0$ and $k_5 = 0$. Then the jump tensor is asymmetric and $N_1 = N_2 = 0$ but $N_3 \neq 0$. We then obtain a unique light cone because (5.1) vanishes identically. However, the closure condition (8.8) is not satisfied, since (8.3)-(8.7) are nontrivial for $N_1 = N_2 = 0$ and $N_3 \neq 0$. This also means that the requirement of a unique light cone does not necessarily implies that χ must be symmetric.

The opposite is also true: Suppose an asymmetric jump tensor (3.5) has the closure property, i.e. (8.8) is fulfilled. Then we find (9.1) again. However, (9.1b) yields $N_2 = -N_3$, and consequently (5.1) together with (4.7) and (4.8) describe the case of a birefringent and dissipative wave propagation.

These examples show that the closure of an *asymmetric* jump (or constitutive) tensor does not guarantee the absence of birefringence, and, vice versa, no-birefringence is not accompanied by the closure property for an asymmetric operator.

10. MOVING ISOTROPIC NONLINEAR MEDIA

Recently, there has been some interest in the light propagating in moving media with nontrivial dielectric and magnetic properties. The first covariant analysis of the Fresnel

equation for this case was done by Kremer [19]. An isotropic medium is characterized by the constitutive law

$$\mathcal{H}^{ij} = \sqrt{|g|} \left[\frac{1}{\mu} F^{ij} + 2 \left(\frac{1}{\mu} - \varepsilon \right) u_k F^{k[i} u^{j]} \right], \quad (10.1)$$

where u^i is the 4-velocity of the moving matter (normalized as usual by $u_i u^i = 1$), and ε and μ are the permeability and permittivity functions of the isotropic medium. The case when they do not depend on the electromagnetic field strength (being constant in space and time, for example) was investigated in [19].

More recently, the nonlinear case when $\varepsilon = \varepsilon(F)$ and $\mu = \mu(F)$ are functions of the electromagnetic field has been studied by De Lorenci et al [14]. However, the attention was restricted to certain special cases, and the general result is still missing.

We can perform a fairly complete analysis of the wave propagation in a nonlinear moving media on the basis of our covariant Fresnel equation (2.10). By differentiation, we easily find the jump tensor (2.7):

$$\begin{aligned} \chi^{ijkl} = \sqrt{|g|} & \left[\frac{2}{\mu} g^{i[k} g^{l]j} + 4 \left(\frac{1}{\mu} - \varepsilon \right) u^{[k} g^{l][i} u^{j]} \right. \\ & \left. + m^{kl} F^{ij} + 2 (m^{kl} - e^{kl}) u_m F^{m[i} u^{j]} \right]. \end{aligned} \quad (10.2)$$

The second line is absent in the linear theory, and the tensors

$$m^{ij} := \frac{\partial(1/\mu)}{\partial F_{ij}}, \quad e^{ij} := \frac{\partial \varepsilon}{\partial F_{ij}}, \quad (10.3)$$

are responsible for the nonlinear electrodynamical effects.

Inspection immediately reveals that the jump tensor density (10.2) contains *both* an axion and a skewon part. There are claims in the literature that axion and skewon, in general, do not have physical sense. However, here we encounter a simple and a physically sound example with the both parts being nontrivial. The irreducible pieces (3.8)-(3.9) read:

$$\begin{aligned} {}^{(1)}\chi^{ijkl} = \sqrt{|g|} & \left[\frac{2}{\mu} g^{i[k} g^{l]j} + 4 \left(\frac{1}{\mu} - \varepsilon \right) u^{[k} g^{l][i} u^{j]} + \frac{1}{2} (m^{ij} F^{kl} + m^{kl} F^{ij}) + (m^{kl} - e^{kl}) P^{[i} u^{j]} \right. \\ & \left. + (m^{ij} - e^{ij}) P^{[k} u^{l]} + \frac{1}{12} \eta^{ijkl} [F^{mn} \tilde{m}_{mn} + 2(\tilde{m}_{mn} - \tilde{e}_{mn}) P^m u^n] \right], \end{aligned} \quad (10.4)$$

$$^{(2)}\chi^{ijkl} = \sqrt{|g|} \left[\frac{1}{2} (F^{ij} m^{kl} - F^{kl} m^{ij}) + (m^{kl} - e^{kl}) P^{[i} u^{j]} - (m^{ij} - e^{ij}) P^{[k} u^{l]} \right], \quad (10.5)$$

and for the axion piece, we have

$$\hat{e}_{ijkl} \chi^{ijkl} = \sqrt{|g|} \hat{e}_{ijkl} [F^{ij} m^{kl} + 2 (m^{kl} - e^{ij}) P^i u^j]. \quad (10.6)$$

Thus, nonlinear isotropic matter does have axion and skewon induced by nonlinearity.

A. Nonmagnetic matter

Let us consider the case when the magnetic constant is independent of the electromagnetic field, that is $m^{ij} = 0$. [In the simplest case, we can restrict the attention to the purely dielectric medium with $\mu = 1$. However, we will formally keep $\mu \neq 1$, for the sake of generality].

It is convenient to make the evident split of (10.2) into the sum $\chi^{ijkl} = \phi^{ijkl} + \psi^{ijkl}$, with

$$\phi^{ijkl} = \sqrt{|g|} \left[\frac{2}{\mu} g^{[k} g^{l]j} + 4 \left(\frac{1}{\mu} - \varepsilon \right) u^{[k} g^{l][i} u^{j]} \right], \quad (10.7)$$

$$\psi^{ijkl} = -2\sqrt{|g|} e^{kl} u_m F^{m[i} u^{j]}. \quad (10.8)$$

It is straightforward to find the Fresnel tensor for the first piece:

$$\mathcal{G}^{ijkl}(\phi) = \frac{\varepsilon}{\mu^2} \text{sign}(g) \sqrt{|g|} \overset{\circ}{g}^{(ij} \overset{\circ}{g}^{kl)}. \quad (10.9)$$

Here we have denoted the so called *Gordon optical metric* [18] as

$$\overset{\circ}{g}^{ij} := g^{ij} + (\varepsilon\mu - 1) u^i u^j. \quad (10.10)$$

Its inverse reads

$$\overset{\circ}{g}_{ij} := g_{ij} + \left(\frac{1}{\varepsilon\mu} - 1 \right) u_i u_j, \quad (10.11)$$

and the determinant can be easily computed: $\det \overset{\circ}{g} = (\det g)/(\varepsilon\mu)$.

Using this optical metric we can simplify the form of (10.7), bringing it to

$$\phi^{ijkl} = \frac{2}{\mu} \sqrt{|g|} \, \overset{\circ}{g}{}^{i[k} \overset{\circ}{g}{}^{l]j} = 2 \sqrt{\frac{\varepsilon}{\mu}} \sqrt{|\overset{\circ}{g}|} \, \overset{\circ}{g}{}^{i[k} \overset{\circ}{g}{}^{l]j}. \quad (10.12)$$

Now, for $\chi = \phi + \psi$, using a compact notation by omitting the indices, we have

$$\mathcal{G}(\chi) = \mathcal{G}(\phi) + \mathcal{G}(\psi) + \frac{1}{4!} (O_1 + O_2 + O_3 + T_1 + T_2 + T_3). \quad (10.13)$$

Here the mixed terms O_a contain one ψ -factor and the T_a 's two ψ -factors. *Postponing the symmetrization over i, j, k, l to the very last moment*, these terms read explicitly as follows:

$$O_1(\phi, \psi, \phi) = \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \phi^{mnri} \psi^{jpsk} \phi^{lqtu}, \quad (10.14)$$

$$O_2(\psi, \phi, \phi) = \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \psi^{mnri} \phi^{jpsk} \phi^{lqtu}, \quad (10.15)$$

$$O_3(\phi, \phi, \psi) = \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \phi^{mnri} \phi^{jpsk} \psi^{lqtu}, \quad (10.16)$$

$$T_1(\psi, \phi, \psi) = \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \psi^{mnri} \phi^{jpsk} \psi^{lqtu}, \quad (10.17)$$

$$T_2(\psi, \psi, \phi) = \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \psi^{mnri} \psi^{jpsk} \phi^{lqtu}, \quad (10.18)$$

$$T_3(\phi, \psi, \psi) = \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \phi^{mnri} \psi^{jpsk} \psi^{lqtu}. \quad (10.19)$$

An important observation is that all T 's are vanishing for (10.8). Indeed, let us denote $P^j := u_i F^{ij}$, then

$$\psi^{ijkl} = -2 \sqrt{|g|} P^{[i} u^{j]} e^{kl}. \quad (10.20)$$

Then we straightforwardly find, for example,

$$\begin{aligned} T_1(\psi, \chi, \psi) &= 4|g| \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} P^{[m} u^{n]} e^{ri} P^{[l} u^{q]} e^{tu} \chi^{jpsk} \\ &= 2|g| \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \chi^{jpsk} P^m u^n (P^l u^q - P^q u^l) e^{ri} e^{sk} = 0. \end{aligned} \quad (10.21)$$

This is zero because either the symmetric $u^n u^q$ or symmetric $P^m P^q$ is contracted with the antisymmetric $\hat{\epsilon}_{mnpq}$. Note that we on purpose write χ^{jpsk} as the second argument, because its form is *arbitrary*, not necessarily equal to (10.7). Analogously, we find:

$$\begin{aligned} T_2(\psi, \psi, \chi) &= 4|g| \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} P^{[m} u^{n]} e^{ri} P^{[j} u^{p]} e^{sk} \chi^{lqtu} \\ &= 2|g| \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \chi^{lqtu} P^m u^n (P^j u^p - P^p u^j) e^{ri} e^{sk} = 0. \end{aligned} \quad (10.22)$$

It is a little bit more nontrivial to prove that T_3 also vanishes. We have, explicitly:

$$\begin{aligned}
T_3(\chi, \psi, \psi) &= 4|g| \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} P^{[j} u^p] e^{sk} P^{[l} u^q] e^{tu} \chi^{mnri} \\
&= |g| \hat{\epsilon}_{mnpq} \hat{\epsilon}_{rstu} \chi^{mnri} (P^j u^p P^l u^q - P^p u^j P^l u^q \\
&\quad - P^j u^p P^q u^l + P^p u^j P^q u^l) e^{sk} e^{tu} = 0.
\end{aligned} \tag{10.23}$$

The first and the last terms in the parentheses contain the symmetric combinations $u^p u^q$ and $P^p P^q$ which are vanishing when contracted with the antisymmetric $\hat{\epsilon}_{mnpq}$. The two remaining terms in the parentheses are reduced, by means of a relabeling of indices, to $-P^p u^q (P^l u^j - P^j u^l)$. Recalling that at the end we impose the symmetrization over the free indices (i, j, k, l) , we thus prove that $T_3(\chi, \psi, \psi) = 0$.

Since in all the three formulas (10.21)-(10.23), the argument χ^{jpsk} is completely arbitrary, we can put $\chi^{jpsk} = \psi^{jpsk}$, in particular. Then (10.21)-(10.23) yields that $\mathcal{G}(\psi) = 0$. As the next choice, we put $\chi^{jpsk} = \phi^{jpsk}$. Then (10.13) combined with (10.21)-(10.23), yields

$$\mathcal{G}(\chi) = \mathcal{G}(\phi) + \frac{1}{4!} (O_1 + O_2 + O_3). \tag{10.24}$$

It thus remains to compute the O -terms. The corresponding calculation is straightforward and simple if we use the representation (10.12). Then we find:

$$O_1(\phi, \psi, \phi) = O_2(\psi, \phi, \phi) = O_3(\phi, \phi, \psi) = -8 \text{sign}(g) \frac{\varepsilon}{\mu} \overset{\circ}{g}^{(ij} \psi^{k|mn|l)} \overset{\circ}{g}_{mn}. \tag{10.25}$$

Note that this result is valid *for all* possible tensors ψ , not only (10.8) which means that we can further use (10.25) for the future calculations involving more general nonlinear pieces (in particular, for the case with a nontrivial m^{ij}). Using then (10.25) in (10.24), we get

$$\mathcal{G}^{ijkl}(\phi + \psi) = \frac{\varepsilon}{\mu^2} \text{sign}(g) \sqrt{|g|} \overset{\circ}{g}^{(ij} \overset{1}{g}^{kl)}. \tag{10.26}$$

Here we denoted

$$\overset{1}{g}^{ij} := \overset{\circ}{g}^{ij} - \frac{\mu}{\sqrt{|g|}} \psi^{(i|mn|j)} \overset{\circ}{g}_{mn}. \tag{10.27}$$

Hence, a purely dielectric nonlinear moving medium will in general exhibit the birefringence effect: the light will propagate in such a medium along the cone of the original optical metric

$\overset{\circ}{g}{}^{ij}$ (one may call it ordinary ray), and along the second cone determined by the metric $\overset{1}{g}{}^{ij}$ (“extraordinary” ray).

The explicit form of the extraordinary optical metric is obtained when we substitute (10.20) into (10.27):

$$\overset{1}{g}{}^{ij} = \overset{\circ}{g}{}^{ij} - \mu P_k e^{k(i} u^{j)} + \frac{1}{\varepsilon} u_k e^{k(i} P^{j)}, \quad (10.28)$$

$$= \overset{\circ}{g}{}^{ij} + \mu e^{k(i} u^{j)} F_{kl} u^l - \frac{1}{\varepsilon} u_k e^{k(i} F^{j)l} u_l. \quad (10.29)$$

Special cases of this general result were considered in [14].

As an example, let us consider a medium in its rest frame. We use adapted coordinates such that $u^i = \delta_0^i = (1, \vec{0})$. We additionally assume that the dielectric permittivity is given by

$$\varepsilon = \bar{\varepsilon} + a \vec{E}^2. \quad (10.30)$$

Here $\bar{\varepsilon}$ and a are constant parameters. The components of the electric vector are defined as usual, $\vec{E} = E_a = -F_{0a}$. Then we find

$$e^{0a} = -e^{a0} = -2a E^a, \quad \text{and} \quad P^j = u_i F^{ij} = (0, E^b). \quad (10.31)$$

The spatial indices are lowered and raised with the help of the 3-metric δ_{ab} (we neglect the gravitational). As a result, from (10.31) we obtain

$$P_k e^{ki} = (-2a \vec{E}^2, \vec{0}), \quad u_k e^{ki} = (0, -2a E^b). \quad (10.32)$$

Finally, using all this in (10.28) we find the components of the extraordinary optical metric:

$$g^{00} = n^2 \left(1 + \frac{2a}{\varepsilon} \vec{E}^2 \right), \quad g^{ab} = -\delta^{ab} - \frac{2a}{\varepsilon} E^a E^b. \quad (10.33)$$

Here $n := \sqrt{\varepsilon\mu}$ is the refraction index of the medium. In this way we obtain a natural description of the optical Kerr effect (see [27,28], for example) when birefringence is induced by the applied electric field.

11. DISCUSSION AND CONCLUSION

In this paper, we have performed a systematic analysis of the light propagation in the nonlinear electrodynamics on the basis of the Fresnel approach. We have considered two classes of models: (a) general nonlinear Lagrangian theories, and (b) moving nonlinear matter. In the former case, the Lagrangian (3.2) is an arbitrary function of the two electromagnetic invariants, whereas in the latter case, the permeability and permittivity functions of the medium (10.1) depend arbitrarily on the electromagnetic field.

The study of the first class of models reveals the *generic* nature of the birefringence effect: The quartic Fresnel surface reduces to the two light cones for *all* nonlinear Lagrangians. We show that the resulting optical metrics are always real and have the correct Lorentzian signature. In this way, we confirm and extend the recent results of [13,14]. Furthermore, we are able to demonstrate the validity of the so called closure–no birefringence conjecture in the context of nonlinear electrodynamics: Birefringence is absent (and thus the quartic Fresnel surface reduces to a unique light cone) if and only if the effective constitutive (or jump) tensor satisfies closure property.

The nonlinear moving matter with the constitutive law (10.1) gives a sound example of a model in which the effective constitutive tensor naturally has nontrivial axion and skewon contributions. Accordingly, one should then expect that the Fresnel surface remains quartic, in general [23]. However, for nonmagnetic material media, we show that birefringence is again the generic effect. The Fresnel surface factorizes into two light cones, one of which corresponds to the Gordon optical metric (independent of nonlinearities), whereas the other (10.28) manifests the nonlinear properties of the model. The optical Kerr effect represents a particular example of our general derivations.

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